MIN-557 **Finite Element Methods** **Autumn 2019-20**

Homework Assignment-1 [Due: August 19, 2019 (during class hours)]

Q1) Use variational calculus to show that the shortest distance curve joining any two points on a plane is a straight line.

Q2) Derive the Euler-Lagrange equations for the case of functional with a single independent variable and two dependent variables, given by

$$I = \int_{x_1}^{x_2} F(y_1, y_1^{'}, y_2, y_2^{'}, x)dx$$

Q3) Based on the results obtained in (2) obtain the optimum functions *x* and *y* that extremize

$$\int_0^1 \left[2x + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right] dt$$

Subject to: $x(0) = 0$, $y(0) = 0$, $x(1) = 1.5$, and $y(1) = 1$.

Q4) For attaining the equilibrium of a circular plate with radius *a* loaded by a force *q(r)*, the following functional has to be minimized,

$$I = D\pi \int_0^a \left[r\left(\frac{d^2w}{dr^2}\right)^2 + \frac{1}{r}\left(\frac{dw}{dr}\right)^2 + 2v\frac{dw}{dr}\frac{d^2w}{dr^2} - \frac{2q}{D}rw \right] dr$$

where *D* and *v* are the elastic constants. Obtain the governing differential equation.

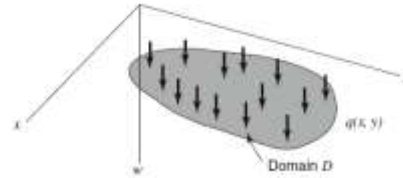
Q5) Consider a more general (three-dimensional) problem of variational calculus wherein a single dependent function *u* and its partial derivatives constitute the functional as given below:

$$\iiint_V F\left(x, y, z, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right) dxdydz$$

where, *x, y, and z* are the independent variables. Derive the Euler-Lagrange equation and arrive the natural and essential boundary conditions.

Q6) Consider a thin membrane of arbitrary shape simply supported on its boundary (i.e., zero displacement on the boundary). The membrane is stretched by a tensile force T which is constant everywhere in the membrane. It is also subjected to a transverse force q which is a function of x and y. Assuming small deformations, show that the static equilibrium is governed by the following differential equation written in terms of the transverse (vertical) displacement (w). Use the energy-based approach to arrive at the differential equation.

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{q}{T}$$



Q7) The total potential energy for a linear elastic body is given by

$$\Pi = \iiint_V U_0 dv - \iiint_V b_i u_i dv - \iint_{S_1} t_i u_i ds$$

Where $U_0 = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$ is the strain energy density function, b_i are the components of body force/unit volume and

t_i are the components of the traction vector applied on the surface S_1. The components of the displacement field are denoted by u_i. Show that the condition of stationary potential energy leads to the equations of stress equilibrium and the statement of the Cauchy's relation $t_i = \sigma_{ji} n_j$ on the traction boundary. Make use of the relation $\frac{\partial U_0}{\partial \varepsilon_{ij}} = \sigma_{ij}$ and the symmetry of the stress tensor in your derivation.